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Grade 6 Math Circles

November 29/30/December 1, 2022

Jeopardy

Jeopardy

Number Systems	Counting	Math Logic	BCC Prep	Set Theory	Geometric Constructions	Pascal's Triangle
100	100	100	100	100	100	100
200	200	200	200	200	200	200
300	300	300	300	300	300	300
400	400	400	400	400	400	400
500	500	500	500	500	500	500

Number Systems - 100

Evaluate the following powers.

a) 3^3

b) 7^0

c) 9^2

Answer

Number Systems - 200

List the bases for the following number systems.

a) Binary

b) Octal

c) Decimal

d) Hexadecimal

Answer

Number Systems - 300

Convert 157_8 to decimal.

Answer

Number Systems - 400

Convert 103_{10} to binary.

Answer

Number Systems - 500

Convert 677_{10} to hexadecimal.

Answer

Number Systems - 100 Answer

a) $3^3 = 27$

b) $7^0 = 1$

c) $9^2 = 81$

Number Systems - 200 Answer

a) Binary: Base 2

b) Octal: Base 8

c) Decimal: Base 10

d) Hexadecimal: Base 16

Number Systems - 300
Answer

$$
\begin{aligned}
157_8 &= (1_8 \times 8^2) + (5_8 \times 8^1) + (7_8 \times 8^0) \\
&= (1 \times 64) + (5 \times 8) + (7 \times 1) \\
&= 64 + 40 + 7 \\
&= 111_{10}
\end{aligned}
$$

Number Systems - 400
Answer

Base 10	$2^6 = 64$	$2^5 = 32$	$2^4 = 16$	$2^3 = 8$	$2^2 = 4$	$2^1 = 2$	$2^0 = 1$	Base 2
103	|	|			|	|	|	1100111
	1	1	0	0	1	1	1	

Number Systems - 500
Answer

Base 10	$16^3 = 4096$	$16^2 = 256$	$16^1 = 16$	$16^0 = 1$	Base 16
677					$2A5$
	0	2	A	5	

Counting - 100

Evaluate $\frac{31!}{29!}$.

Answer

Counting - 200

How many ways can the letters $EASY$ be rearranged?

Answer

Counting - 300

How many 4-digit positive integers have no repeated digits?

Answer

Counting - 400

Given 10 (distinct) points on a plane,
no three of which lie on the same line,
how many lines pass through 2 of these points?

Answer

Counting - 500

How many 3-digit positive integers are divisible by at least one of 2, 3, or 5?

Answer

Counting - 100
Answer

930

Counting - 200
Answer

$$4! = 24$$

Counting - 300
Answer

$$9 \times 9 \times 8 \times 7 = 4536$$

Counting - 400
Answer

45

Counting - 500
Answer

660

Math Logic - 100

- I will eat either pizza or spaghetti for dinner.
- I will not eat spaghetti for dinner.

What would I have for dinner?

Answer

Math Logic - 200

What is the mathematical symbols for “or”, “and”, ”not”, in that order?

Answer

Math Logic - 300

Make a truth table for $P \vee Q$.

Answer

Math Logic - 400

Find the simplest equivalent logical expression of $\neg(\neg\neg P \vee \neg Q) \wedge Q$.

Answer

Math Logic - 500

Four boys, Joshua, Daniel, Nicholas, and Ryan, are at home to watch some movies. Who does like Action movies?

1. Joshua is at one of the ends.
2. The boy wearing the Black shirt is somewhere to the left of the youngest boy (11-year-old).
3. Joshua likes Horror movies.
4. The 14-year-old boy is at the third position.
5. The boy wearing the Red shirt is somewhere between the 13-year-old boy and the one who likes Action movies, in that order.
6. Daniel likes Thriller movies.
7. The boy who is going to eat Cookies is at one of the ends.
8. The boy wearing the Black shirt is exactly to the left of the one who likes Thriller movies.
9. The boy who is going to eat Crackers is exactly to the right of the boy who likes Comedy movies.
10. The boy wearing the Red shirt is somewhere between the boy who is going to eat Popcorn and Nicholas, in that order.
11. At one of the ends is the boy who likes Thriller movies.
12. Nicholas is somewhere between Joshua and Daniel, in that order.
13. At the first position is the boy wearing the Green shirt.

Answer

(Taken from https://www.brainzilla.com/)

Math Logic - 100
Answer

Pizza!

Math Logic - 200
Answer

$\vee, \wedge, \neg$

Math Logic - 300 Answer

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F

Math Logic - 400 Answer

$$\neg P \wedge Q$$

Math Logic - 500 Answer

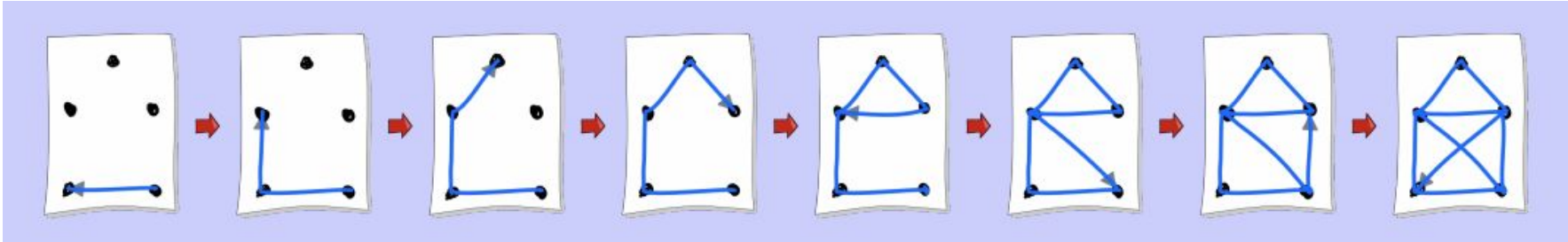
Nicholas

BCC Prep - 100

Connect the Dots (BCC Grade 7/8 2020)

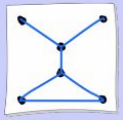
Zhi likes to draw. He creates his pictures by drawing dots and then connecting them with line segments in one motion, never picking up his pencil and never drawing the same line segment twice.

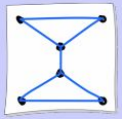
This is how Zhi draws a picture of a house:



Which of the following pictures can Zhi draw?

(A) 

(B) 

(C) 

(D) 

Answer

BCC Prep - 200

Movie Theatre Seats (BCC Grade 7/8 2019)

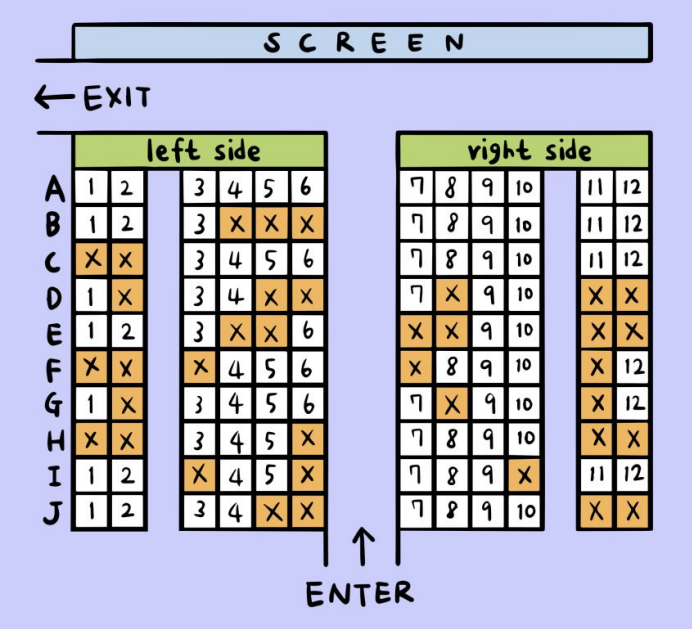
Three friends Alex, Bao, and Chiti are choosing seats in a movie theatre. The seats marked X can't be selected because someone else has already taken them.

Alex, Bao, and Chiti each say what will make them happy:

- Alex: "I want to sit on the right side."
- Bao: "The three of us must sit right beside each other without any seats or aisles between us."
- Chiti: "I don't like when the screen is too close! Let's not sit in the first three rows."

For example, if they choose seats G3, G4, and G5, then Alex will be unhappy. If they choose D7, D9, and D10, then Bao will be unhappy. If they choose A7, A8, and A9, then Chiti will be unhappy.

Answer



In how many ways can the three friends choose seats so that they are all happy?

(A) 3 (B) 6 (C) 7 (D) 9

BCC Prep - 300

Library Books (BCC Grade 7/8 2020)

Beavertown Library has only a small pile of books. When a beaver wishes to borrow a book, they take the book that is on the top of the pile and record their name. When a beaver returns a book, they place their book on the top of the pile and record their name again.

At the beginning of the week the pile of books was arranged as shown:	The library's records at the end of the week show the following information:
	

Which book did Cato borrow?

(A) Charlotte's Web

(B) Curious George

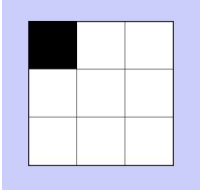
(C) Go, Dog, Go!

(D) The Hobbit

Answer

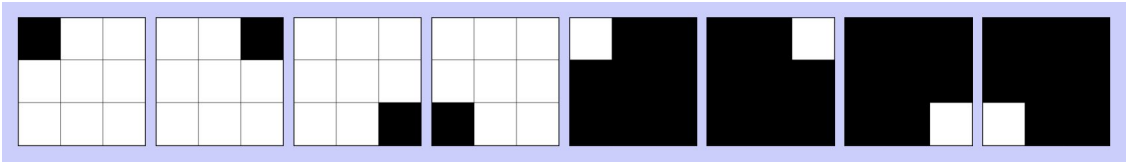
BCC Prep - 400

What is THIS? (BCC Grade 7/8 2017)



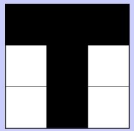
Beatrice Beaver is playing around with her simple 3-by-3 computer screen. She can paint some squares black. For example, if she painted only the top-left square, the screen would look like this:

Her computer also has "rotate" and "invert" buttons. The "rotate" button rotates the screen clockwise by 90 degrees. The "invert" button changes all white squares to black and all black squares to white. For example, when Beatrice presses the "rotate" and "invert" buttons after painting only the top-left square, she can create a total of eight different patterns:

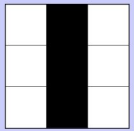


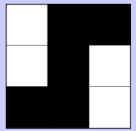
Beatrice begins with different images on the screen. She uses the two types of buttons any number of times and in any order trying to make different patterns.

Which of the following starting images allows Beatrice to make the largest number of different patterns?

(A) 

(B) 

(C) 

(D) 

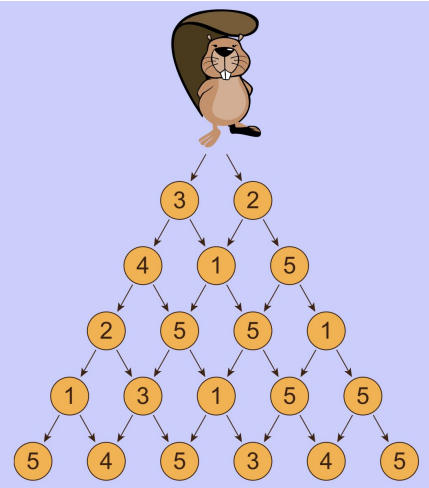
Answer

BCC Prep - 500

Collecting Wood (BCC Grade 7/8 2016)

During his descent from the mountain top, the beaver, Theseas, is collecting wood for his lodge from several stations. Every station holds a different amount of wood. While he is descending, he cannot change direction and start climbing again, that is, he can only go in the directions of the arrows.

The paths between stations are given in the image below. Every circle is a station and the number in the circle represents the amount of wood available at that station.

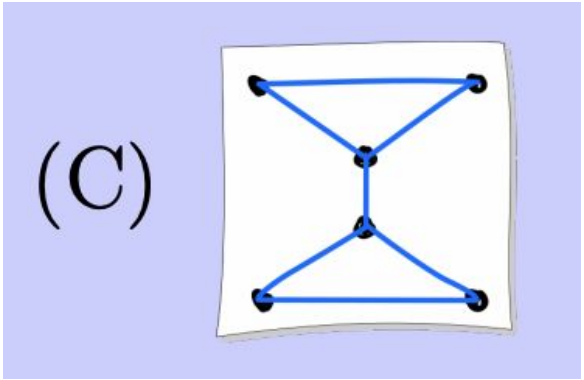


What is the maximum total amount of wood that Theseas can collect during his descent?

(A) 19 (B) 20 (C) 21 (D) 22

Answer

BCC Prep - 100 Answer



BCC Prep - 200 Answer

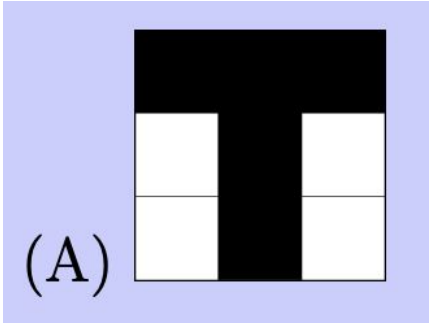
(B) 6

Jeopardy

BCC Prep - 300
Answer

(B) Curious George

BCC Prep - 400 Answer



BCC Prep - 500 Answer

(C) 21

Interactive link

Jeopardy

Set Theory - 100

Let $A = \{1, 2, 3\}$ and $B = \{1, 3, 5\}$ what is $A \cup B$?

Answer

Set Theory - 200

Draw a Venn Diagram for 3 sets, A, B, and C.

Answer

Set Theory - 300

Rewrite the set definition of $A = \{1, 2, 3, 4, 5, 6, 7\}$ by using the elementhood test.

Answer

Set Theory - 400

Each element belongs to A or B or both. Given

$$|A \cap B| = 5$$
$$|A| = 17$$
$$|B| = 15$$

what is the total number of elements, $|A \cup B|$?

Answer

Set Theory - 500

Power Set

Given a set A, we define the **power set** of A to be the set

$$\mathbb{P}(A) = \{X \mid X \subseteq A\}.$$

In words, the power set of A, $\mathbb{P}(A)$, is a collection of all subsets of A including $\varnothing$ and A itself.

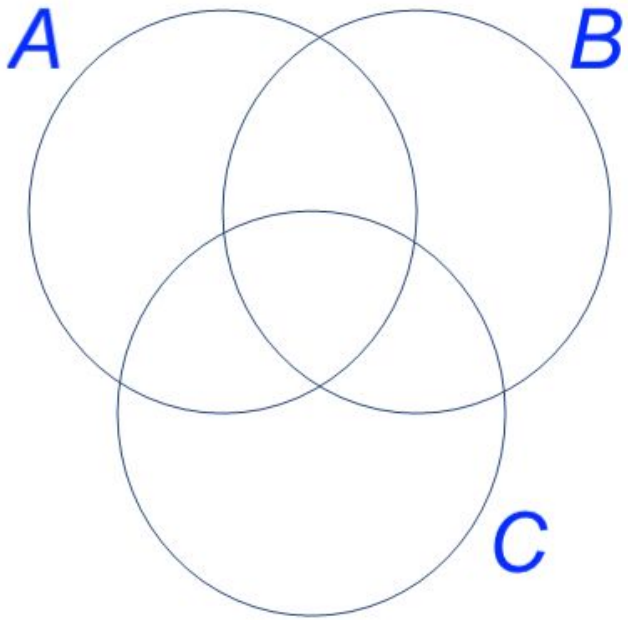
Let $A = \{1, 2, 3, 4, 5\}$. What is $|\mathbb{P}(A)|$?

Answer

Set Theory - 100
Answer

$$\{1, 2, 3, 5\}$$

Set Theory - 200 Answer



Set Theory - 300
Answer

$$A = \{x \mid x \text{ is a natural number less than or equal to } 7\}$$

Set Theory - 400
Answer

27

Set Theory - 500
Answer

$$2^5 = 32$$

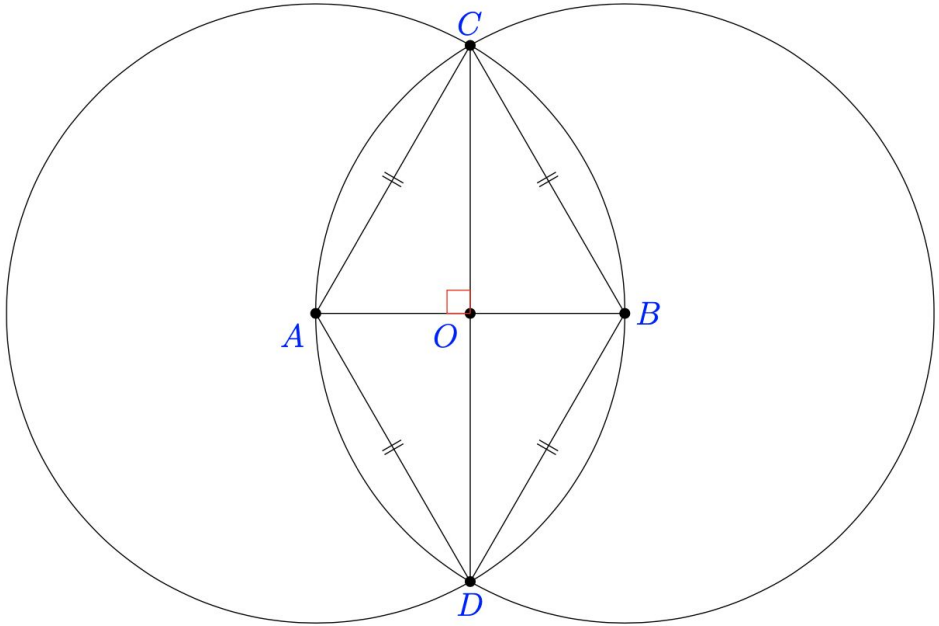
Geometric Constructions - 100

Identify the two tools we use to create geometric constructions.

Answer

Geometric Constructions - 200

In the following diagram, name an angle which has measure 30°.



Answer

Geometric Constructions - 300

Given a line segment $\overline{AB}$, describe how to construct a line segment of length $4 \times AB$.

Answer

Geometric Constructions - 400

Let ABC be an equilateral triangle, O its circumcenter, and I its incenter. True or False:

a) $OA = OB = OC$

b) O and I are the same point.

c) The line passing through O and A is perpendicular to $\overline{BC}$.

Answer

Geometric Constructions - 500

Describe how to construct a square.

You may use the construction of a perpendicular bisector without listing out the steps

Answer

Geometric Constructions - 100
Answer

Compass and straightedge.

Geometric Constructions - 200 Answer

$\angle DCA$

Geometric Constructions - 300 Answer

1. Extend the line which passes through A and B. Let's call it ℓ.
2. Construct the circle with center A and radius AB, let it intersect ℓ again at C.
3. Construct the circle with center C and radius BC, let it intersect ℓ again at D.
4. The line segment $\overline{CD}$ has length $4 \times AB$.

Geometric Constructions - 400 Answer

a) True.

b) True.

c) True.

Geometric Constructions - 500 Answer

1. Construct a line segment $\overline{AB}$.
2. Construct the perpendicular bisector of $\overline{AB}$ and call it ℓ. Let ℓ intersect $\overline{AB}$ at O.
3. Draw a circle with center O and radius OA. Let it intersect ℓ at C and D.
4. $ACBD$ is a square.

Pascal's Triangle - 100

List at least two other names for Pascal's triangle.

Answer

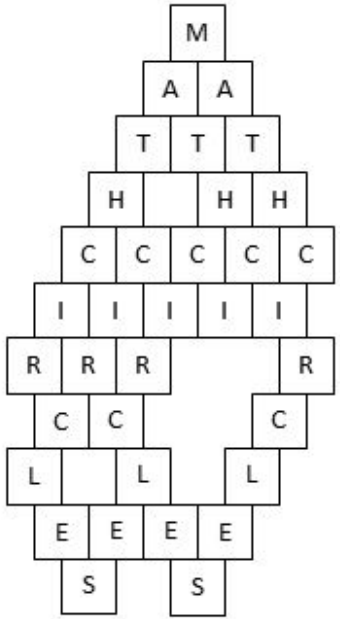
Pascal's Triangle - 200

What is the sum of the entries in row 19 of Pascal's triangle?

Answer

Pascal's Triangle - 300

Using the following diagram, how many ways can you make a path that spells MATH CIRCLES by starting at M and moving downwards?



Answer

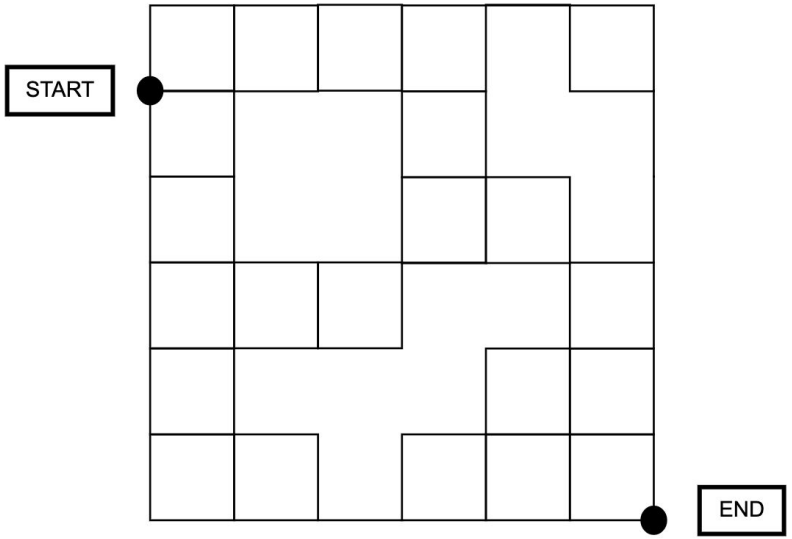
Pascal's Triangle - 400

What is the entry that has row number 20 and term number 18?

Answer

Pascal's Triangle - 500

For the grid below, count how many paths can be taken from the dot marked START to the dot marked END by moving along the lines and only moving down or to the right.



Answer

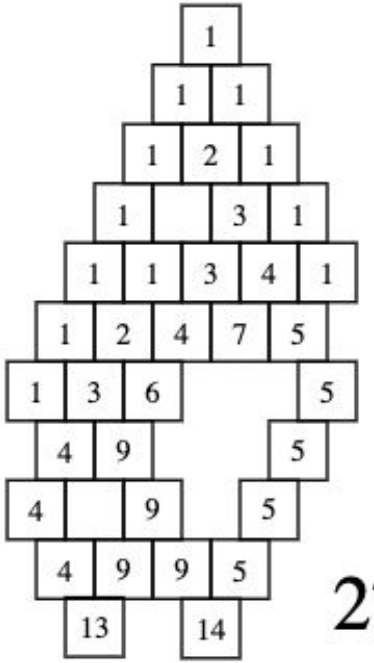
Pascal's Triangle - 100 Answer

- Staircase of Mount Meru
- Khayyam Triangle
- Yang Hui's Triangle
- Tartaglia's Triangle
- Figurate Triangle
- Combinatorial Triangle
- Binomial Triangle

Pascal's Triangle - 200 Answer

$$2^{19} = 524288$$

Pascal's Triangle - 300 Answer



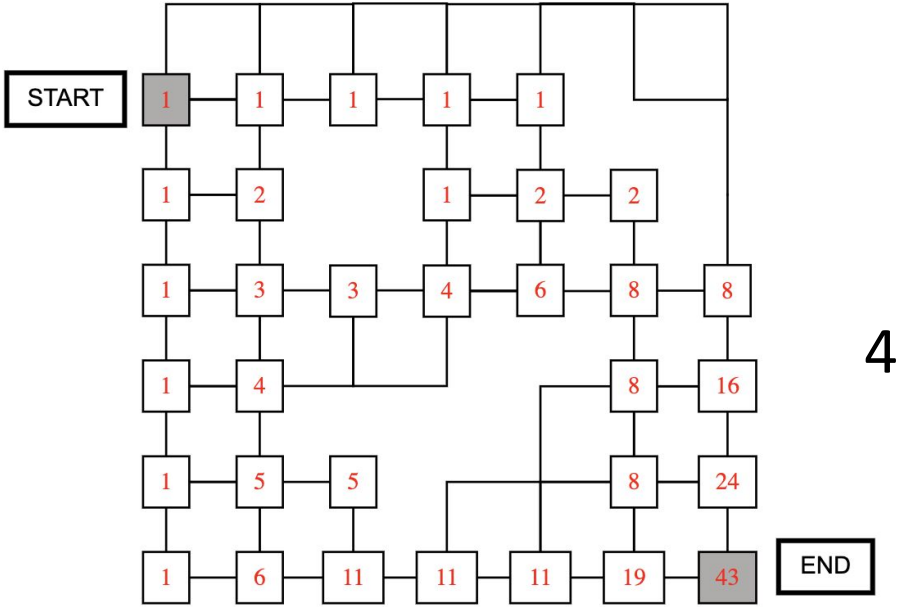
27 ways

Jeopardy

Pascal's Triangle - 400 Answer

$$\binom{20}{18} = \frac{20!}{18!(20-18)!} = \frac{20\times19\times18!}{18!\times2!} = \frac{20\times19}{2} = 190$$

Pascal's Triangle - 500 Answer



43 paths

Jeopardy